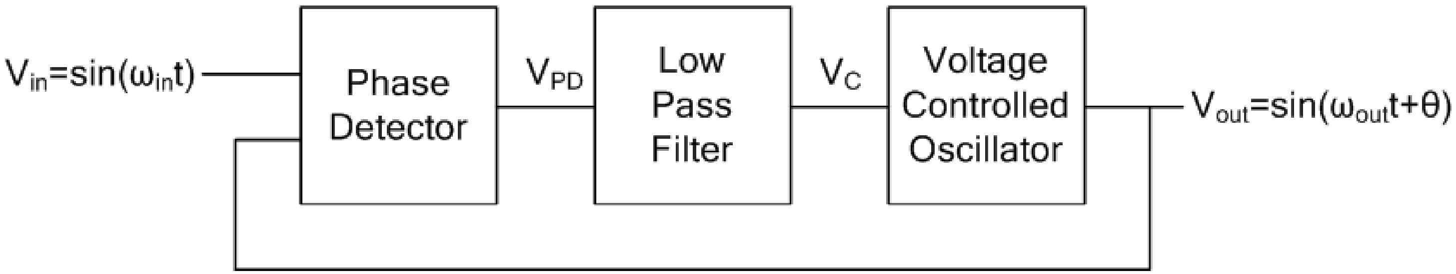
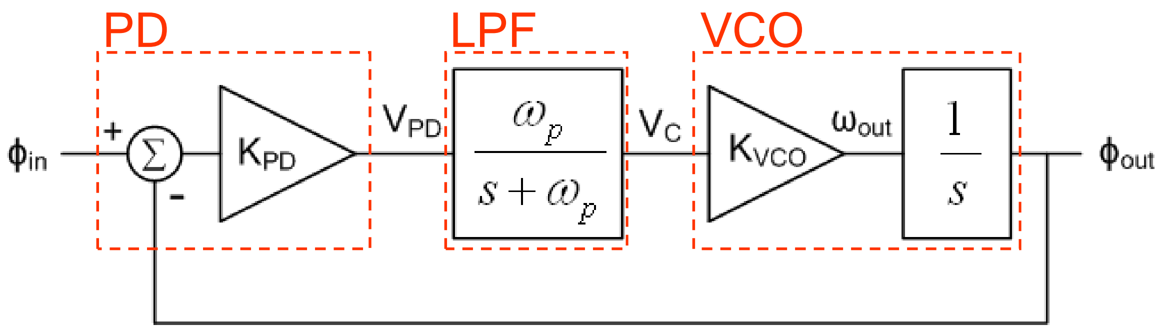
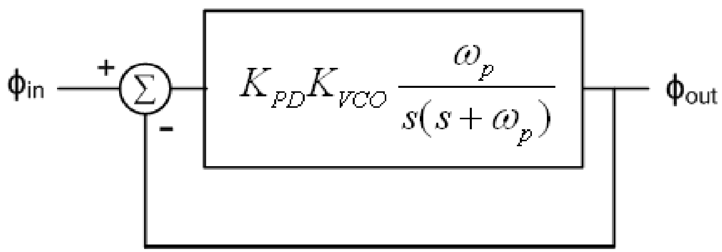
Lect. 23: PLL Dynamics

PLL Block Diagram



Linear Model





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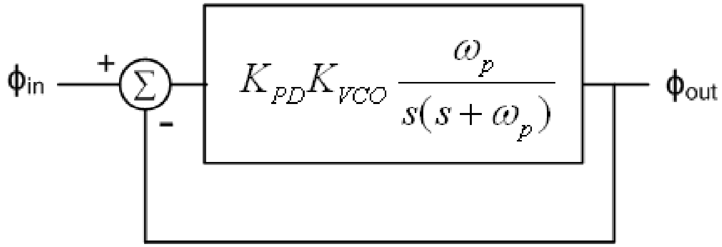

Open loop gain:

$$G(s) = K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}$$

Closed loop gain

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{G(s)}{1+G(s)} = \frac{K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}}{1+K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$

➔ 2nd order LPF!

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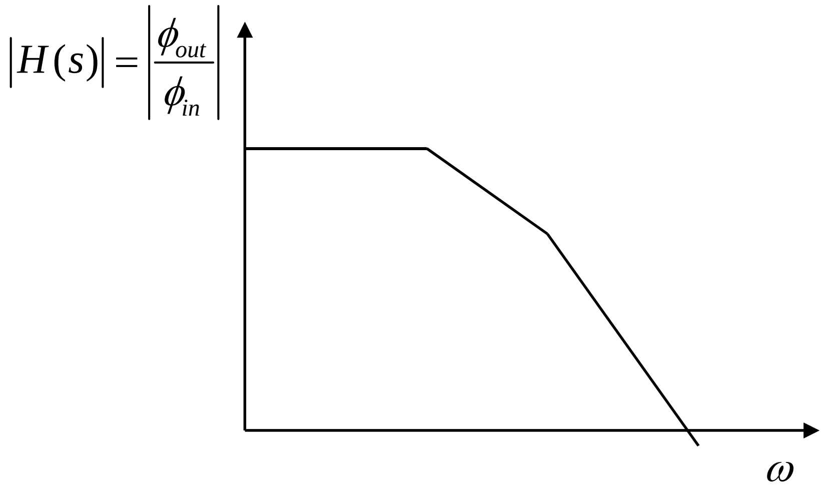
$$H(s)=\frac{\phi_{out}}{\phi_{out}}=\frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$



Note that input and output are *'phase'*.

What does ω mean in x-axis?

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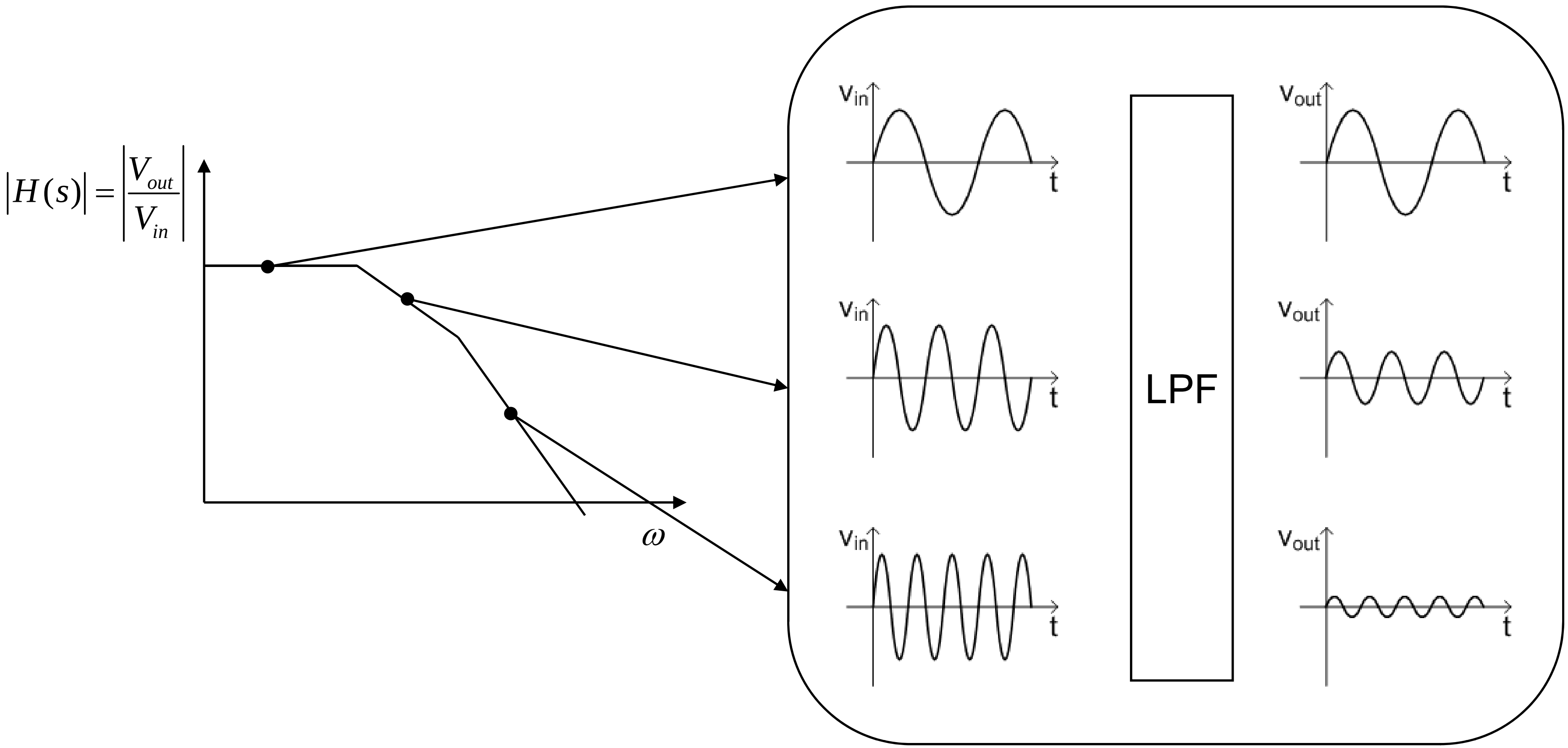
In LPF,



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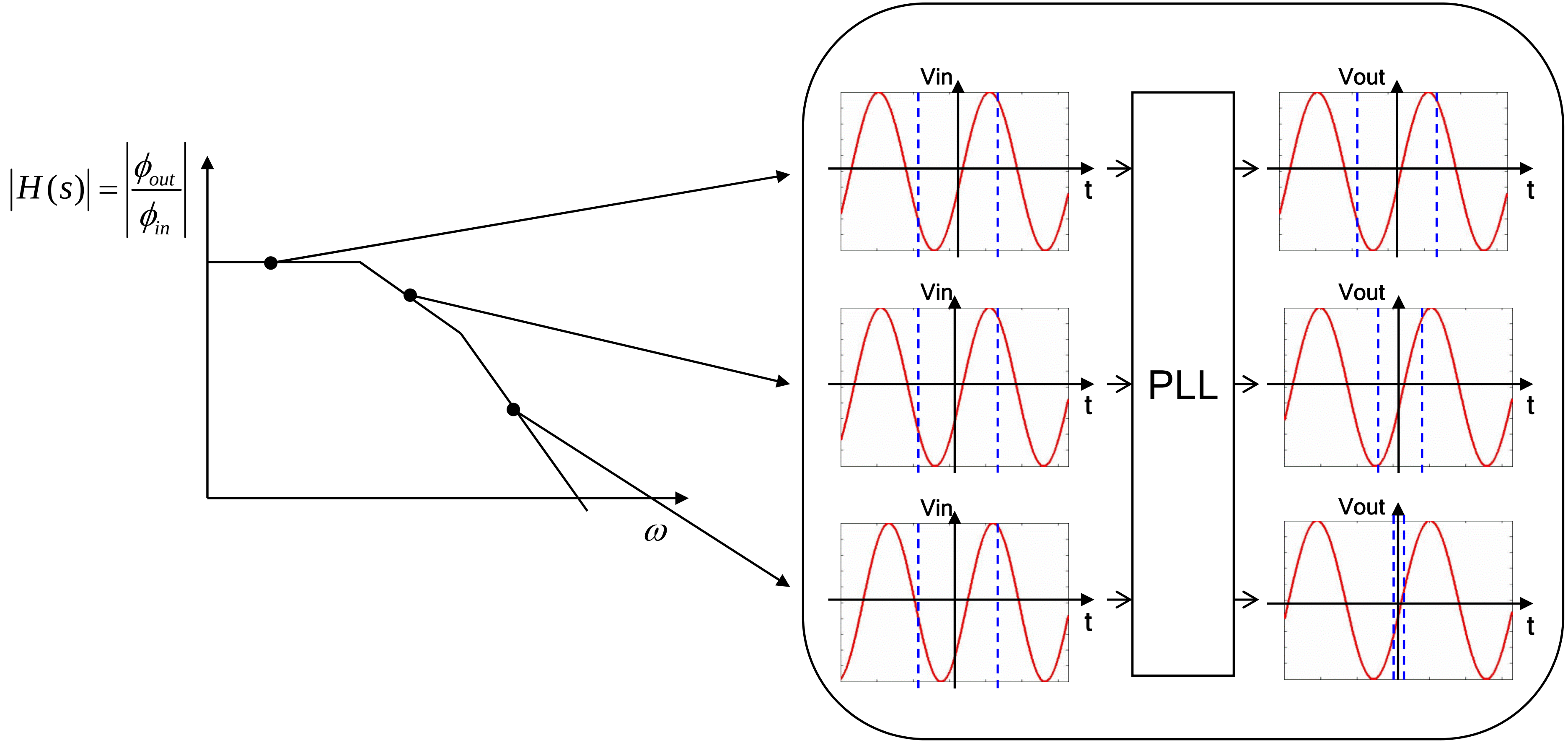
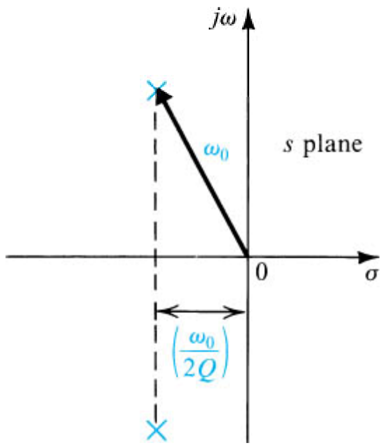
In PLL,



Lect. 23: PLL Dynamics

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{K_{PD} K_{VCO} \omega_p}{s^2 + \omega_p s + K_{PD} K_{VCO} \omega_p}$$



2nd order system
$$\boldsymbol{H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q) s + \omega_0^2}}$$

$$\omega_0 = \sqrt{\omega_P K_{PD} K_{VCO}}$$

Use ω_n, natural frequency ($\omega_n = \omega_0$)

$$Q = \sqrt{\frac{K_{PD} K_{VCO}}{\omega_P}}$$

Use damping factor $\zeta = \frac{1}{2Q} = \sqrt{\frac{\omega_P}{K_{PD} K_{VCO}}}$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\omega_{out}}{\omega_{in}} = \frac{s\phi_{out}}{s\phi_{in}} = \frac{\phi_{out}}{\phi_{in}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

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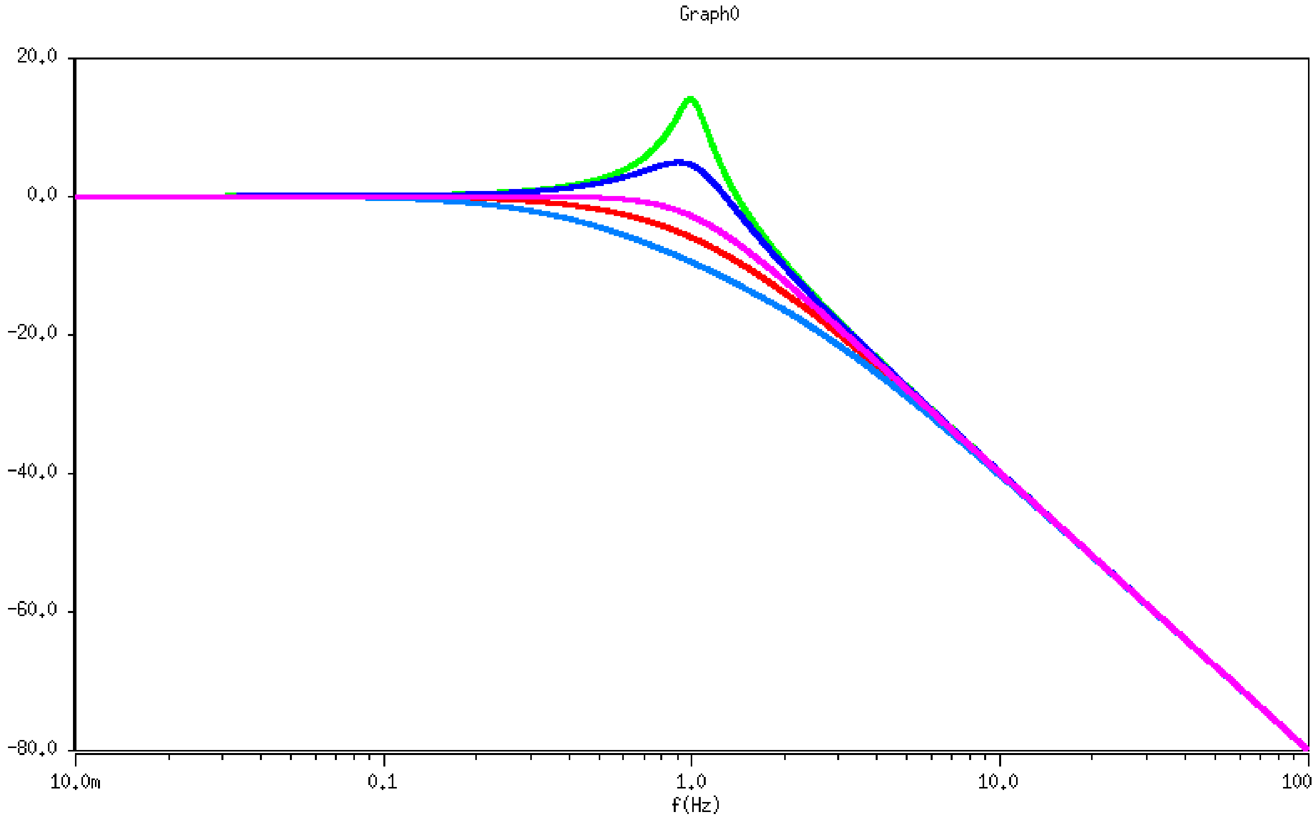
Damping factor dependece $H(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$



$\omega_n = 2\pi$

ξ = 0.1

ξ = 0.3

ξ = 0.7

ξ = 1

ξ = 1.5

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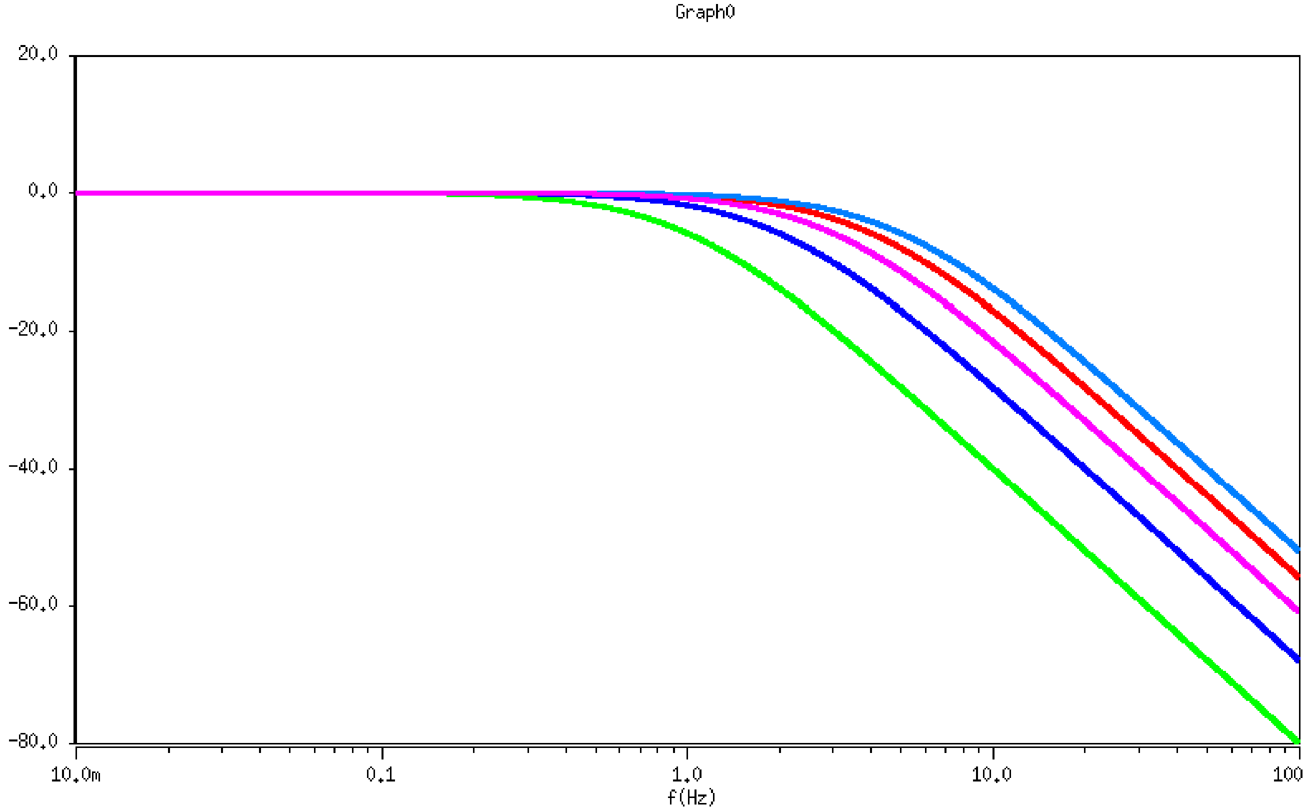
Natural frequency dependence

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



ξ= 1

$\omega_n = 2\pi \times 1$

$\omega_n = 2\pi \times 2$

$\omega_n = 2\pi \times 3$

$\omega_n = 2\pi \times 4$

$\omega_n = 2\pi \times 5$

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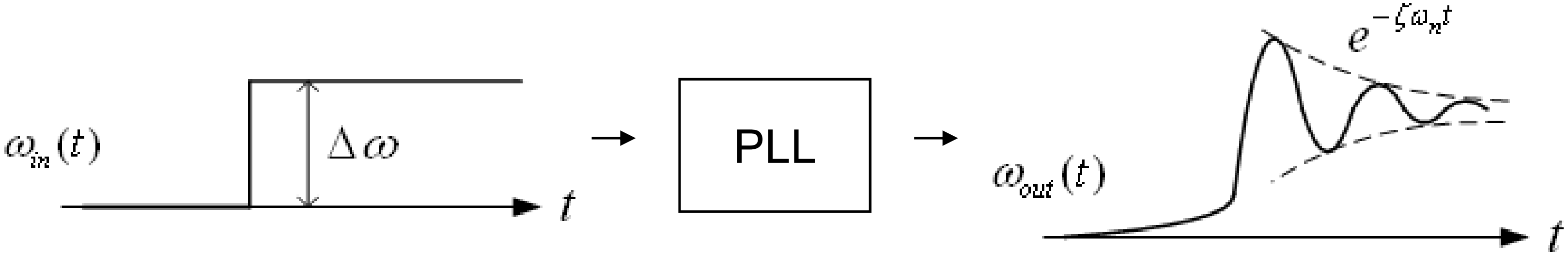
Step response

$$\omega_{in}(t) = \Delta\omega \cdot u(t)$$

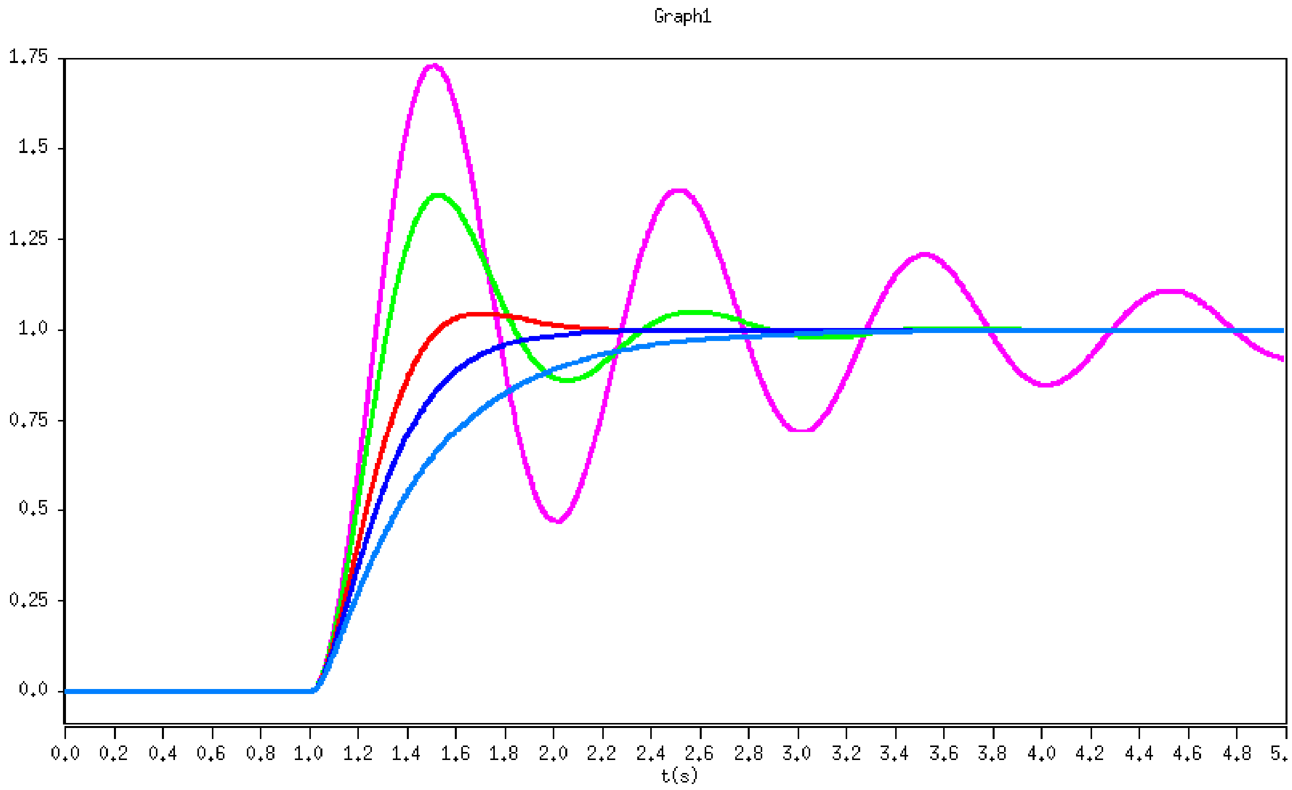


$$\omega_{out}(t) = \left\{1 - e^{-\zeta\omega_n t}[\cos(\omega_n\sqrt{1-\zeta^2}\cdot t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\cdot t)]\right\}\Delta\omega \cdot u(t)$$

Lect. 23: PLL Dynamics

Damping factor dependence: step response

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$\omega_n = 2\pi$

$\xi = 0.1$

$\xi = 0.3$

$\xi = 0.7$

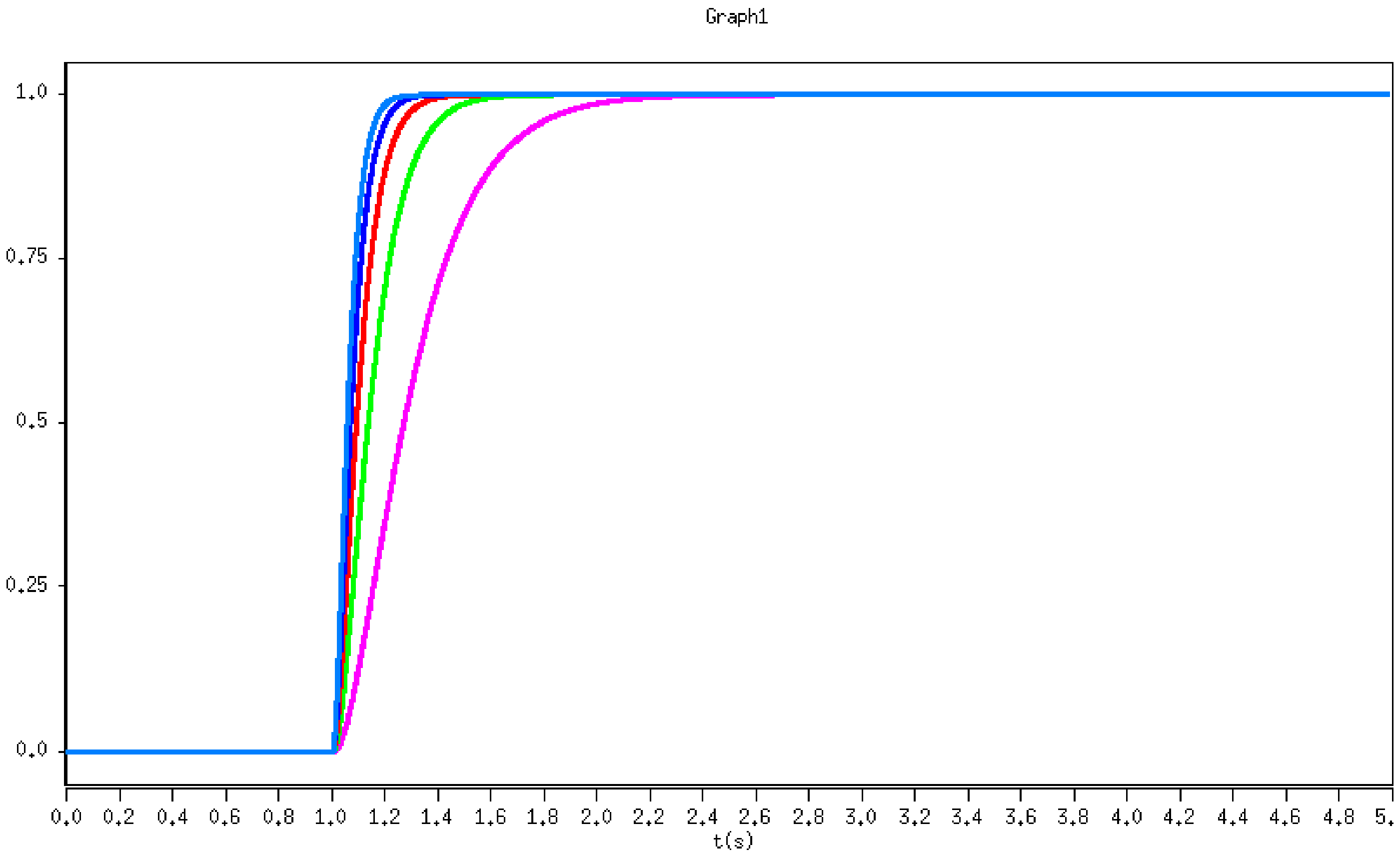
$\xi = 1$

$\xi = 1.5$

Lect. 23: PLL Dynamics

Natural frequency dependence: Step Response

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$\xi = 1$

$\omega_n = 2\pi \times 1$

$\omega_n = 2\pi \times 2$

$\omega_n = 2\pi \times 3$

$\omega_n = 2\pi \times 4$

$\omega_n = 2\pi \times 5$